

Ideas on charm and beauty decays

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Abstract

The parton model, a zero-order approximation in many treatments, is shown to be a “semiclassical” model whose results for certain averages also hold (correspondence principle) in quantum mechanics. Algebraic techniques developed for the Mössbauer effect exploit simple features of commutators to obtain sum rules showing the validity of the parton model for $b \rightarrow c$ semileptonic decays in the classical limit, $\hbar \rightarrow 0$, where all commutators vanish, and in general, even when binding effects are included, for the lowest moments of the lepton energy spectrum at fixed 3-momentum transfer. Interference between the $u\bar{u}$ and $d\bar{d}$ components of the ρ^0 and ω wave functions can be used as clues to contributions from small weak amplitudes and CP violation in decays to final states including these vector mesons.

1 Parton model (Mössbauer) sum rules for $b \rightarrow c$ decays

We assume that bound states of one heavy quark and other degrees of freedom are described by a Hamiltonian depending upon the heavy quark flavour only via its mass. The dynamics of the other degrees of freedom, including their interactions with the heavy quark, are described by a flavour-independent operator ΔH , which depends on the heavy quark co-ordinate \vec{X} and on the other degrees of freedom, denoted by ξ_ν , but not on the heavy quark momentum \vec{P} . Thus $[\Delta H, \vec{X}] = 0$, but $[\Delta H, \vec{P}] \neq 0$ and we can write the Hamiltonian H_Q for systems containing a single heavy quark of flavour $Q = b$ or c .

$$H_Q = H(\vec{P}, m_Q, \vec{X}, \xi_\nu) = \sqrt{m_Q^2 + \vec{P}^2} + \Delta H; \quad (Q = b, c) \quad (1.1)$$

The hadronic transition in semileptonic $b \rightarrow c$ decays is described by the matrix element $\langle f_c | J(\vec{q}) | i_b \rangle$ of the fourier component carrying three-momentum (\vec{q}) of the flavour-changing weak current between an initial state $|i_b\rangle$ containing one and only one valence b quark and a final state $|f_c\rangle$ containing one and only one valence c quark. We assume that $J(\vec{q})$ depends only on \vec{X} , normalize the current and define moments of the final state energy distribution to obtain

$$[J(\vec{q}), \Delta H] = 0; \quad \sum_{|f_c\rangle} |\langle f_c | J(\vec{q}) | i_b \rangle|^2 = \langle i_b | J^\dagger(\vec{q}) J(\vec{q}) | i_b \rangle = 1 \quad (1.2)$$

$$\begin{aligned} \langle [E_c(\vec{q})]^n \rangle &\equiv \sum_{|f_c\rangle} (E_c)^n |\langle f_c | J(\vec{q}) | i_b \rangle|^2 = \langle i_b | J^\dagger(\vec{q}) (H_c)^n J(\vec{q}) | i_b \rangle = \\ &= \langle i_b | J^\dagger(\vec{q}) \left\{ \sqrt{m_c^2 + \vec{P}^2} + \Delta H \right\}^n J(\vec{q}) | i_b \rangle \end{aligned} \quad (1.3)$$

These assumptions hold in a number of conventionally used models, and in particular in the nonrelativistic constituent quark potential models with various potentials. Spin effects are neglected; they are taken into account in a more detailed treatment[1].

The information about the other degrees of freedom ξ_ν in the moments (1.3) appears in the operator ΔH and disappears when ΔH acts directly either to the left or to the right on the initial state $|i_b\rangle$.

$$\Delta H |i_b\rangle = (H_b - \sqrt{m_b^2 + \vec{P}^2}) |i_b\rangle = \left(M_i - \sqrt{m_b^2 + \vec{P}^2} \right) |i_b\rangle \quad (1.4)$$

where M_i is the the eigenvalue of H_b in the initial state $|i_b\rangle$ and is just the mass of this state.

Eqs. (1.3-1.4) express the basic physics of this approach. In any model satisfying the assumptions (1.1-1.2) the moments (1.3) with $n \leq 2$, where all the ΔH factors can be moved either to the left or to the right so that they act on $|i_b\rangle$, are expressed as expectation values in the initial state of single-particle operators which depend only upon the dynamical variables of the heavy quark and are determined completely by the one-particle density matrix for the heavy quark in the initial state. They are the same as the results for a naive parton model whose parton distribution is given by this one-particle density matrix. The case $n = 0$ is just the Bjorken sum rule which effectively states that the heavy quark lifetime is independent of binding except for phase space factors.

Only in the moments for $n \geq 3$ where commutators of the form $[\Delta H, \vec{P}]$ appear do deviations from parton results occur. These are proportional to commutators which vanish in the classical limit where $\hbar \rightarrow 0$.

Explicitly, for $n = 1$ and $n \geq 2$

$$\langle [E_c(\vec{q})] \rangle = \langle i_b | J^\dagger(\vec{q}) \sqrt{m_c^2 + \vec{P}^2} J(\vec{q}) + \{M_i - \sqrt{m_b^2 + \vec{P}^2}\} J^\dagger(\vec{q}) J(\vec{q}) | i_b \rangle \quad (1.5a)$$

$$\begin{aligned} \langle [E_c(\vec{q})]^n \rangle &= \langle i_b | \left(\{M_i - \sqrt{m_b^2 + \vec{P}^2}\} J^\dagger(\vec{q}) + J^\dagger(\vec{q}) \sqrt{m_c^2 + \vec{P}^2} \right) (H_c)^{(n-2)} \\ &\quad \cdot \left(J(\vec{q}) \{M_i - \sqrt{m_b^2 + \vec{P}^2}\} + \sqrt{m_c^2 + \vec{P}^2} \cdot J(\vec{q}) \right) | i_b \rangle \end{aligned} \quad (1.5b)$$

The sum rules for $n=1$ and $n=2$ are simplified[1] by expressing $J(\vec{q})$ explicitly in terms of \vec{X} , the heavy quark mass difference $\delta m \equiv m_b - m_c$, and the free recoil energy $R(\vec{q})$ and the “isomer” or “isotope” shift I_{bc} , defined respectively as

$$R(\vec{q}) \equiv H[(\vec{P} + \vec{q}), m_c, \vec{X}, \xi_\nu] - H[\vec{P}, m_c, \vec{X}, \xi_\nu] = \sqrt{(\vec{P} + \vec{q})^2 + m_c^2} - \sqrt{\vec{P}^2 + m_c^2} \approx \frac{q^2}{2m_c} \quad (1.6a)$$

$$I_{bc} \equiv \delta m + H[\vec{P}, m_c, \vec{X}, \xi_\nu] - H[\vec{P}, m_b, \vec{X}, \xi_\nu] \approx \vec{P}^2 \cdot \frac{\delta m}{2m_c m_b} \quad (1.6b)$$

$$\langle [E_c(\vec{q})]^n \rangle = \langle i_b | \{M_i + R(\vec{q}) + I_{bc} - \delta m\}^n | i_b \rangle \quad (1.6c)$$

where \approx denotes the nonrelativistic approximation. These sum rules can also be written for the energy E_W carried by the W; i.e. by the leptons,

$$\begin{aligned} \langle E_W(\vec{q}) \rangle &\equiv \sum_{|f_c\rangle} E_W | \langle f_c | J(\vec{q}) | i_b \rangle |^2 = M_i - \langle [E_c(\vec{q})] \rangle = \delta m - \langle i_b | R(\vec{q}) + I_{bc} | i_b \rangle \\ &\approx \delta m - \frac{q^2}{2m_c} - \langle i_b | \vec{P}^2 | i_b \rangle \cdot \frac{\delta m}{2m_c m_b} \end{aligned} \quad (1.7a)$$

$$\langle [E_W(\vec{q})]^2 \rangle - \langle [E_W(\vec{q})] \rangle^2 = \langle i_b | \{R(\vec{q}) + I_{bc}\}^2 | i_b \rangle - \langle i_b | \{R(\vec{q}) + I_{bc}\} | i_b \rangle^2 \approx$$

$$\approx \frac{\langle i_b | \vec{P}^2 | i_b \rangle \cdot q^2}{3m_c^2} + \frac{(\delta m)^2}{4m_c^2 m_b^2} \cdot (\langle i_b | P^4 | i_b \rangle - \langle i_b | P^2 | i_b \rangle^2) \quad (1.7b)$$

An upper bound for the transition to a given final state $|f_m\rangle$ with energy E_m is obtained by replacing all energies except E_m in the sum rule with the lowest possible energy $E_g = M_D + \frac{q^2}{2M_D}$; the energy of the lowest available state of the charmed system,

$$E_m |\langle f_m | J(\vec{q}) | i_b \rangle|^2 + E_g (1 - |\langle f_m | J(\vec{q}) | i_b \rangle|^2) \leq \langle [E_c(\vec{q})] \rangle = M_i + R(\vec{q}) + I_{bc} - \delta m$$

$$\approx M_i + \frac{q^2}{2m_c} + \langle i_b | \vec{P}^2 | i_b \rangle \cdot \frac{\delta m}{2m_c m_b} - \delta m \quad (1.8a)$$

$$|\langle f_m | J(\vec{q}) | i_b \rangle|^2 \leq \frac{\langle [E_c(\vec{q})] \rangle - E_g}{E_m - E_g} \approx \frac{1}{E_m - E_g} \cdot \left(\frac{q^2}{2M_D m_c} \cdot [M_D - m_c] + \epsilon \right) \quad (1.8b)$$

where

$$\epsilon \equiv [M_i - m_b] - [M_D - m_c] + \langle i_b | \vec{P}^2 | i_b \rangle \cdot \frac{\delta m}{2m_c m_b} = \langle i_b | H_c | i_b \rangle - M_D \quad (1.8c)$$

The matrix element $\langle i_b | H_c | i_b \rangle$ gives a value for M_D exact to first order in the perturbation $H_c - H_b$ and in the reciprocal mass difference $\frac{m_b - m_c}{m_b m_c}$. Thus ϵ is second order in $1/m_c$.

Thus the probability of excitation by an energy $E_m - E_g$ is bounded by the ratio to this energy of the small energy $\frac{q^2}{2M_D m_c} \cdot [M_D - m_c]$ which goes to zero as $q^2 \rightarrow 0$ with a small correction ϵ which vanishes in the heavy quark symmetry limit. This treatment can be extended to include spin and relativistic effects. However it can be expected to be already particularly good in the low-recoil domain of small q^2 where the bound (1.8) places serious limits on the probability of high excitations; i.e. on low lepton energies.

2 Use of $\rho - \omega$ Interference Effects in B and D Decays

The ρ^0 and ω are equal mixtures with opposite relative phase of the $u\bar{u}$ and $d\bar{d}$ vector quarkonium states, which we denote respectively by V_u and V_d . Thus if ρ^0 and ω are produced via a quark diagram leading to a single flavour state, either V_u or V_d , they should both be produced equally[2] with a definite relative phase and show the interference effect originally suggested by Glashow [3] and subsequently extensively observed experimentally [4]. These predictions are particularly sensitive via interference to small contributions from other diagrams

producing the state forbidden in the dominant diagram[5]. Similar effects have been considered for decays into $\eta - \eta'$ modes [6].

The relation between production and decay processes of the ρ^0 and ω can be understood by a comparison with the $K - \bar{K}$ system. The four “quark-flavour” vector meson eigenstates $\rho^+(u\bar{d})$, $V_d(d\bar{d})$, $V_u(u\bar{u})$ and $\rho^-(d\bar{u})$ are directly analogous to the four kaon quark-flavour eigenstates: $K^+(u\bar{s})$, $K^0(d\bar{s})$, $\bar{K}^0(s\bar{d})$ and $K^-(s\bar{u})$. In both cases the two neutral states are nearly degenerate and both quark-flavour eigenstates can decay into two pions or into three pions.

The decay interaction mixes the quark flavour eigenstates into short-lived mesons K_S and V_S (or ρ^0), which decay dominantly into two pions, and long-lived mesons K_L and V_L (or ω), which decay dominantly into three pions. The decay eigenstates are both eigenstates to a very good approximation of a symmetry, CP for the kaons and G-parity for the vectors, which forbids the 2π decay for K_L and ω . However because both CP and G are broken by relatively small effects both the K_L and ω have a small 2π branching ratio and interesting interference effects are observed.

Neutral kaons are produced and leave the production vertex as flavour eigenstates K^0 and \bar{K}^0 . They decay after leaving the range of all final state interactions as equal mixtures of K_L and K_S with opposite relative phases. If neutral vector mesons are similarly produced as flavour eigenstates and decay only after leaving the range of all final state interactions, they decay as V_u and V_d ; i.e as equal mixtures of ρ^0 and ω with opposite relative phases. This leads to interesting experimental consequences which can be useful for investigations of weak interactions and CP violation. However the lifetimes here are much shorter and the escape from the range of final state interactions before decay is open to question.

Good experimental evidence that the vector mesons do decay outside the range of final state interactions was first noted in strong interaction reactions described by diagrams where a final state with one vector flavour eigenstate is forbidden by the Alexander-Zweig [2] or OZI rule; giving a selection rule and predicting the equality of the two observable cross sections,

$$\sigma(K^-p \rightarrow \Lambda V_d) = 0; \quad \sigma(K^-p \rightarrow \Lambda\omega) = \sigma(K^-p \rightarrow \Lambda\rho^0) \quad (2.1)$$

This prediction from the implied ρ^0 and ω production via V_u was confirmed by experiment and the $\rho - \omega$ interference subsequently observed [7]. Final state interactions are expected to be very different for the $\Lambda\omega$ and $\Lambda\rho^0$ states since they have different isospins and are coupled to completely different hadronic channels. Thus the experimentally observed equality (2.1) is evidence that the decay occurs outside the range of final state interactions.

In weak interactions there are a number of ways to test whether the decay occurs outside the range of final state interactions. Final state interactions

should be directly observed in final states involving ρ mesons as a perturbation of the Breit-Wigner shape of the decay pion spectrum. In semileptonic decays to ρ^0 and ω where there are no final state interactions, the ρ^0 and ω should be produced equally from the produced flavour eigenstate, with a relative phase measurable by interference.

Weak interaction diagrams tend to produce the ρ and ω via only their V_d or V_u components since the quark lines in these diagrams have definite flavour labels. In the $B^+ \rightarrow K^+ \rho^0$ and $B^+ \rightarrow K^+ \omega$ decays $K^+ V_u$ is produced by both the Cabibbo-suppressed color-favored and Cabibbo-suppressed color-suppressed spectator tree diagrams and also by other diagrams like the penguin which first produce a $\bar{s}u$ intermediate state and then produce the additional $q\bar{q}$ via gluons

$$B^+(\bar{b}u) \rightarrow_{(cstree)} (\bar{u}u\bar{s})u \rightarrow K^+ V_u; \quad B^+(\bar{b}u) \rightarrow_{(penguin)} \bar{s}u \rightarrow K^+ V_u \quad (2.2)$$

Production of $K^+ V_d$ is OZI forbidden both for the penguin diagram (2.2) and for diagrams producing a $\bar{d}d$ pair by final state interactions following the tree diagram. The OZI rule forbids all processes where both members of a $q\bar{q}$ pair produced by gluons end up in the same final state hadron. In this particular case the production of $K^+ V_d$ is also forbidden by flavour SU(3), even without assuming OZI, for all transitions via an intermediate $\bar{s}u$ state. A spin-zero $K^+ V_d$ state has exotic flavour quantum numbers and cannot couple to a single quark-antiquark pair. This most easily seen by noting the exotic flavour quantum numbers $J^{PG} = 0^{++}$ of the $\pi^+ \phi$ state related to $K^+ V_d$ by the SU(3) transformation which exchanges d and s flavours. By analogy with (2.1) we obtain

$$BR(B^+ \rightarrow K^+ V_d) = 0 \quad BR(B^+ \rightarrow K^+ \omega) = BR(B^+ \rightarrow K^+ \rho) \quad (2.3)$$

This prediction can be checked directly by experiment and the same $\rho - \omega$ interference observed in the strong reaction (2.1)[7] should also be observed here.

The interference is observable in detailed analysis of the $\pi^+ \pi^-$ spectrum over the mass range of the ρ resonance. The isospin violating $\omega \rightarrow \pi^+ \pi^-$ has a branching ratio of 2.2%. The width of the ω is 8.4 MeV while that of the ρ is 149 MeV. Thus if the ρ and ω are produced equally in any reaction or decay, the $\pi^+ \pi^-$ decay mode seen at the omega peak will come from both the ρ and the ω and the relative intensities of the two contributions is given by:

$$\frac{I_\omega(B \rightarrow \pi^+ \pi^- X)}{I_\rho(B \rightarrow \pi^+ \pi^- X)} \approx 0.022 \cdot \frac{149}{8.4} \approx 0.39 \quad (2.4)$$

If the two contributions are coherent, the total contribution is given by

$$\frac{I_{total}(B \rightarrow \pi^+ \pi^- X)}{I_\rho(B \rightarrow \pi^+ \pi^- X)} \approx (1 + \sqrt{0.39} \cos \alpha)^2 = 1 + 1.25 \cos \alpha + 0.39 \cos^2 \alpha \quad (2.5)$$

where α is the relative phase of the ρ and ω contributions.

If these predictions are confirmed experimentally, the same approach can be used for the more interesting case of $B^o \rightarrow K^o \rho^o$ and $B^o \rightarrow K^o \omega$ decays, where the Cabibbo-suppressed color-suppressed spectator tree diagram again produces V_u but the penguin diagram and all other diagrams which go via an intermediate $\bar{q}q$ pair produce V_d . Tree production of $K^o V_d$ and penguin production of $K^o V_u$ are both OZI and SU(3) forbidden. Thus

$$B^o(\bar{b}d) \rightarrow_{(cstree)} (\bar{u}u\bar{s})d \rightarrow K^o V_u; \quad B^o(\bar{b}d) \rightarrow_{(penguin)} \bar{s}d \rightarrow K^o V_d \quad (2.6)$$

$$\frac{BR(B^o \rightarrow K^o \rho^o)}{BR(B^o \rightarrow K^o \omega)} = \left| \frac{T+P}{\bar{T}-\bar{P}} \right|^2 = \left| 1 + \frac{2P}{\bar{T}-\bar{P}} \right|^2 \approx 1 + 4Re(P/\bar{T}) \quad (2.7a)$$

$$\frac{BR(\bar{B}^o \rightarrow \bar{K}^o \rho^o)}{BR(\bar{B}^o \rightarrow \bar{K}^o \omega)} = \left| \frac{\bar{T}+\bar{P}}{T-P} \right|^2 = \left| 1 + \frac{2\bar{P}}{T-P} \right|^2 \approx 1 + 4Re(\bar{P}/T) \quad (2.7b)$$

where T , P , \bar{T} and \bar{P} denote respectively the contributions to the decay amplitudes (2.7a) and to the charge conjugate decay amplitudes (2.7b) from tree and penguin diagrams.

This offers the possibility of detecting the penguin contribution and also measuring the relative phase of penguin and tree contributions, as well as detecting CP violation in a difference between the charge-conjugate ρ/ω ratios (2.7a) and (2.7b). The relations (2.7) provide additional input from $B \rightarrow K\omega$ decays that can be combined with isospin analyses of $B \rightarrow K\rho$ decays to separate penguin and tree contributions [8]. A similar additional input is obtainable from combining ω decay modes with isospin analyses of other ρ decay modes[9]

In the Cabibbo-favored $B^o \rightarrow \bar{D}^o \rho^o$ and $B^o \rightarrow \bar{D}^o \omega$ decays, the color-suppressed spectator tree diagram produces V_d but the W-exchange diagram and all other diagrams which go via an intermediate $\bar{c}u$ pair now produce V_u in the transitions allowed by OZI,

$$B^o(\bar{b}d) \rightarrow_{(cstree)} (\bar{c}u\bar{d})d \rightarrow \bar{D}^o V_d; \quad B^o(\bar{b}d) \rightarrow_{(Wexc)} \bar{c}u \rightarrow \bar{D}^o V_d \quad (2.8)$$

$$\frac{BR(B^o \rightarrow \bar{D}^o \rho^o)}{BR(B^o \rightarrow \bar{D}^o \omega)} = \left| \frac{T+W}{\bar{T}-\bar{W}} \right|^2 = \left| 1 + \frac{2W}{\bar{T}-\bar{W}} \right|^2 \approx 1 + 4Re(W/\bar{T}) \quad (2.9)$$

where T , and W denote contributions to decay amplitudes from tree and W-exchange diagrams respectively. Here both tree and W-exchange involve the same combination of CKM matrix elements. Thus no CP-violating relative phase is expected.

In Cabibbo suppressed decays into charmonium and ρ or ω e.g. $B^o \rightarrow \psi \rho^o$ and $B^o \rightarrow \psi \omega$, the color-suppressed spectator tree diagram produces V_d , but the W-exchange diagram and all other diagrams which go via an intermediate

$\bar{c}c$ pair cannot produce a single charmonium state in transitions allowed by OZI. Thus only the tree can contribute and

$$B^o(\bar{b}d) \rightarrow (\bar{c}d\bar{d}) \rightarrow \psi V_d; \quad BR(B^o \rightarrow \psi \rho^o) = BR(B^o \rightarrow \psi \omega) \quad (2.10)$$

and the definite relative phase for production via V_d is predicted for $\rho - \omega$ interference.

One interesting case where data are already available [10] is in the charm decays

$$BR[D^o(c\bar{u}) \rightarrow \bar{K}^o \rho^o] = (6.1 \pm 3.0) \times 10^{-3}; \quad BR[D^o(c\bar{u}) \rightarrow \bar{K}^o \omega] = (2.5 \pm 0.5)\% \quad (2.11a)$$

$$BR[D^o(c\bar{u}) \rightarrow \bar{K}^o \phi] = (8.8 \pm 1.2) \times 10^{-3} \quad (2.11b)$$

The dominant diagram for the decays (2.11) is expected to be the color-suppressed spectator tree diagram which gives the V_u component of the ρ and ω , and predicts equal branching ratios for the two decays (2.11a) and zero for the ϕ .

$$D^o(c\bar{u}) \rightarrow_{(cstree)} (s\bar{u}d)\bar{u} \rightarrow \bar{K}^o V_u \quad (2.12a)$$

The data therefore indicate the presence of another contribution.

The decay can also proceed via a color-favored spectator tree diagram followed by a final state charge exchange rescattering via the intermediate state $K^- \rho^+$. The final state interaction can be expected to be enhanced if there is a K^* resonance in this mass region. The ρ^o and ω are produced via the V_d component and the ϕ can also be produced.

$$D^o(c\bar{u}) \rightarrow_{(cftree)} (s\bar{u}d)\bar{u} \rightarrow \bar{K}^- \rho^+ \rightarrow \bar{K}^{*o}(s\bar{d}) \rightarrow \bar{K}^o V_d \quad (2.12b)$$

$$D^o(c\bar{u}) \rightarrow_{(cftree)} (s\bar{u}d)\bar{u} \rightarrow \bar{K}^- \rho^+ \rightarrow \bar{K}^{*o}(s\bar{d}) \rightarrow \bar{K}^o \phi \quad (2.12c)$$

The V_u component is not produced by this mechanism since the decay $\bar{K}^{*o}(s\bar{d}) \rightarrow \bar{K}^o V_u$ is OZI and SU(3) forbidden.

In all cases the V_u or V_d can hadronize into ρ^o or ω and equal magnitudes are predicted for the ρ and ω final states. However, the relative phase is opposite in the two cases. The measurement of this phase in interference experiments can distinguish between the two mechanisms (2.12a) and (2.12b) and check the validity of assumptions regarding color suppression in tree diagrams and the role of final state interactions. There should be significant peaks and dips observed in the $\pi^+ \pi^-$ spectrum in heavy meson decay modes involving the ρ and ω .

The $\pi^+ \pi^-$ spectrum for the color-suppressed spectator tree contribution (2.12a) where the ρ and ω are produced via the $u\bar{u}$ component should be similar to that observed [7] in strangeness exchange reactions with K^- beams like (2.1). If, however, the color-favored transition via an intermediate resonance (2.12b)

is dominant the interference should have the exact opposite sign. The same interference should be observed in the Cabibbo-suppressed decay (2.12d) which also goes via V_d .

Other contributions to the decays (2.12) with very different flavour properties are

$$D^o(c\bar{u}) \rightarrow_{(W_{exc})} s\bar{d} \rightarrow \bar{K}^o V_d; \quad D^+(c\bar{d}) \rightarrow_{(Ann)} u\bar{d} \rightarrow \pi^+ \rho \quad (2.13)$$

The W-exchange decay to $\bar{K}^o V_u$ is OZI and SU(3) forbidden and the simple annihilation diagram without gluon emission from the initial state is forbidden for the $\pi^+ \omega$ state by G-parity. Note that the $\pi^+ \omega$ final state has the exotic quantum numbers $J^{PG} = 0^{-+}$ and cannot be produced by strong interactions from an intermediate state containing only a single quark-antiquark pair.

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